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GEOMETRY.

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187. Proposed by G. TUCKER, M. A.

AD, BE, CF are the altitudes of the triangle ABC; k_1, k_1'; k_2, k_2'; k_3, k_3' are the S points of the triangles EAB, FCA; FBC, DCB; DCA, EBC, respectively; prove that $k_3'k_1=k_4'k_2=k_2'k_3=R\sin A\sin B\sin C$. ρ_1, ρ_1'; ρ_2, ρ_2'; $\rho_3\rho_3'$ are the Brocard radii of the above triangles, prove that (1) $\rho_1\rho_2\rho_3=\rho_1'\rho_2'\rho_3'$; (2) $(\rho_2'^2-\rho_3^2)/a^2+(\rho_3'^2-\rho_1^2)/b^2+(\rho_1'^2-\rho_2^2)/c^2=\frac{3}{64}$; (3) the sets of 4 Brocard-points for the above pairs of triangles are concyclic (on three circles); (4) the tangents from any one of the right angles of the above triangles to the Brocard circle of the tangent is a mean proportional between the tangents to the same circle from the remaining (two) angles.

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The S point of each triangle is found by drawing from its right angle a line perpendicular to its opposite side, and taking its mid-point. Let k_1, k_2, and k_3 be the mid-points of HF, FK, and DI, respectively, and k_1', k_2', k_3', the mid-points of GE, LD, and JE, respectively. From the inscribed quadrilateral $BGEJ$ we get $GB.JE+BJ.GE=GJ.BE$, $JE\sin A+GE\sin C=GJ$.

$\therefore$ $\sin A\sin C(a\cos C+c\cos A)=GJ=b\sin A\sin C$.

Similarly, $KH=c\sin A\sin B$, $LI=a\sin B\sin C$.

$\therefore$ $GJ=KH=LI=2R\sin A\sin B\sin C$.

$\therefore$ $k_3'k_1=\frac{1}{2}GJ=k_1'k_2=\frac{1}{2}KH=k_2'k_3=\frac{1}{2}LI=R\sin A\sin B\sin C$.

(1) Let ω_1, ω_1', m_2, ω_2', ω_3, ω_3' be the Brocard angles; then $\omega_1=\omega_1'$, $\omega_2=\omega_2'$, $\omega_3=\omega_3'$, for $\cot\omega_1=\cot\omega_1'=\cot A+\tan A$.

$\therefore$ $b\rho_1=c\rho_1'=\frac{1}{4}bc(1-3\cos^2A\sin^2A)^{\frac{1}{2}}$, $c\rho_2=a\rho_2'=\frac{1}{4}ac(1-3\cos^2B\sin^2B)^{\frac{1}{2}}$, $a\rho_3=b\rho_3'=\frac{1}{4}ab(1-3\cos^2C\sin^2C)^{\frac{1}{2}}$.

$\therefore$ $\rho_1\rho_2\rho_3=\rho_1'\rho_2'\rho_3'$, $(\rho_2'^2-\rho_3^2)a^2+(\rho_3'^2-\rho_1^2)b^2+(\rho_1'^2-\rho_2^2)c^2=0$.

$$(2)\ (\rho_2'^2-\rho_3^2)/a^2+(\rho_3'^2-\rho_1^2)/b^2+(\rho_1'^2-\rho_2^2)/c^2=\frac{c^2-b^2}{16a^2}+\frac{a^2-c^2}{16b^2}$$
$$+\frac{b^2-a^2}{16c^2}-\frac{3\triangle^4}{a^2b^2b^2}\left[\frac{c^2-b^2}{a^4}+\frac{a^2-c^2}{b^4}+\frac{b^2-a^2}{c^4}\right].$$

(3) Let M, M_1 be the Brocard-points of EAB; M_2, M_3 those of FCA. Since $\omega_1=\omega_1'$, AM_3M and AM_1M_2 are three each on a straight line.

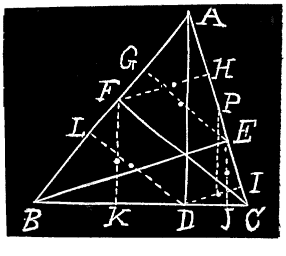
But $AM=c\sin\omega_1/\sin A$, $AM_3=b\sin\omega_1\cot A$,

$AM_2=b\sin\omega_1/\sin A$, $AM_1=c\sin\omega_1\cot A$.

$\therefore$ $AM.AM_3=AM_1.AM_2$ or $AM:AM_2=AM_1:AM_3$.

$\therefore$ M, M_1, M_2, M_3 are concyclic.

Similarly for the remaining pairs of triangles.



(4) Let t, t_1, t_2 be the tangents from D, C, A to the Brocard circle.

Then $t=(DI.Dk_3)^{\frac{1}{2}}=\frac{1}{\sqrt{2}}b\sin\frac{1}{2}C\cos\frac{1}{2}C$,

$$t_1=(CP.CI)^{\frac{1}{2}}=\frac{1}{\sqrt{2}}b\cos C,\ t_2=(AP.AI)^{\frac{1}{2}}=\frac{1}{\sqrt{2}}b\sin C.$$

$\therefore\ t^2=t_1t_2.\quad \therefore\ t_1:t=t:t_2.$

Similarly for the other triangles.

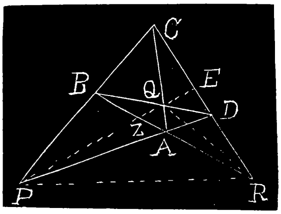
Also proved by *J. R. HITT*.

188. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

ABCD is a quadrilateral whose diagonal triangle is *PQR*, *P* on *AD* and *R* on *AB*. *PQ* meets *AB* in *Z*. If *C* moves along *PB* what will happen to *Z*?

Solution by J. R. HITT, A. M., Coronal Institute, San Marcos, Tex., and G. W. GREENWOOD, A. M., McKendree College, Lebanon, Ill.

Let the figure be drawn as indicated in the proposition. Now, the sides *PQ*, *PR*, of the diagonal triangle, and the opposite sides *AD*, *BC*, of the quadrilateral, form a harmonic pencil. The three rays, *PR*, *PD*, *PC*, remain fixed as *C* moves along *BC*; hence, ray *PE* remains fixed. Therefore, *Z* is fixed, since *RB* is fixed.



Also demonstrated by *G. B. M. ZERR*.

189. Proposed by J. C. CORBIN, Pine Bluff, Ark.

The perpendicular from the right angle on the hypotenuse of a right-angled-triangle is a harmonic mean between the segments of the hypotenuse made by the point of contact of the inscribed circle. [From Casey's *Sequel to Euclid.*]

Solution by LON C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, and J. E. SANDERS, Hackney, Ohio.

Denote by *ABC* the right-angled-triangle, whose sides opposite the angles *A*, *B*, *C* are *a*, *b*, *c*, respectively, where *C* is the right angle.

Let *r* be the radius of the incircle with center *O*, which is tangent to *AB* at the point *G*. Draw *CD* perpendicular to *AB*, *OE* to *BC*, and *OF* to *AC*. Then by the condition of the problem, we have

$$CD=\frac{2BG.GA}{BG+GA}\ldots.(1).$$

Since $AG=AF=b-r=\frac{1}{2}[c-(a-b)]$, and $BG=BE=a-r=\frac{1}{2}[c+(a-b)]$, the right member of (1) reduces to

$$\frac{c^2-(a-b)^2}{2c}=\frac{ab}{c}\ldots.(2).$$

Now from the similar right triangles *BDC* and *BCA*, we have